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ON THE ANALYSIS OF STRIP-PLOT EXPERIMENTS

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Abstract

In some agricultural experiments, applying treatment to the experimental units requires certain method due to the nature of treatment type. Strip-plot experiment is the one of that designs. This paper will discuss several methods of analyzing such design, especially in calculating the sum of squares : Classical Sigma, QR Decomposition, and Partitioned Design Matrices.

Keywords: *sum of squares, classical sigma, QR decomposition, partitioned design matrices, strip-plot experiment*

1. INTRODUCTION

In many experiments where a factorial arrangement is desired, it may not be possible to randomize completely the order of experimentation. There are many practical situations in which it is not all feasible to even randomize within a block. Under certain conditions these restrictions will lead to a split-plot design [**1**]. The modification of the simple split-plot design goes by a variety of names : "Split-plot in strips" [**2**], "two-way whole plots", "subunits in strips", "strip-plot", "split-block" [**3**] and so on. The Strip-Plot designs are used primarily in agricultural experiments. In the most basic setting, there are two factors, say A and B. Factor A is applied to whole plots as in the simple Split-Plot design. But then factor B is applied to "whole plots" (or "strips") which are orthogonal to the whole plots of factor A. [**4**].

As an illustration, factor A might be irrigation systems; each usually require a large amount of land for the experimental units. Factor B might be herbicide spraying which ordinarily would be applied with equipment to large area of land to avoid excessive turning, crop damage, and so on. Thus, the levels of these two factors would need to be applied orthogonal to each other to keep from confounding A and B effects.

2. STRIP-PLOT MODEL

The linear model for the Basic Strip-Plot Model is :

$$Y_{ijk} = \mu + \rho_i + \alpha_j + \delta_{ij} + \beta_k + \eta_{ik} + (\alpha\beta)_{jk} + \varepsilon_{ijk}$$

where μ is the overall mean, ρ_i the i-th replicate effect $i=1,2,...,r$, α_j the effect of j-th level of factor A $j=1,2,...,a$, δ_{ij} the error component of factor A, β_k the effect of k-th level of factor B $k=1,2,...,b$, η_{ik} the error component of factor B, $(\alpha\beta)_{jk}$ the interaction component for j-th level of A and k-th level of B, and ε_{ijk} the residual component or the error componen for the AB interaction.

For inference purposes, the following assumptions are made : (i) the δ*'s* are i.i.d. N$(0,\sigma_\delta^2)$, (ii) the η*'s* are i.i.d. N$(0, \sigma_\eta^2)$, (iii) the ε*'s* are i.i.d. N$(0,\sigma_\varepsilon^2)$, (iv). the δ*'s*, η*'s* and ε*'s* are distributed independently of each other. Additional assumptions are made about the treatment components, depending upon their fixed or random nature.

The **Classical Sigma** notation to calculate the Sum of Squares is as follows :

$$\text{CF = Correction Factor} = \left(\sum_{i,j,k} Y_{ijk}\right)^2 \Big/ rab$$

$$\text{SS[Total]} = \sum_{i,j,k} Y_{ijk}^2 - CF$$

$$\text{SS[Reps]} = \frac{1}{ab}\sum_i Y_{i..}^2 - CF$$

$$\text{SS[A]} = \frac{1}{rb}\sum_j Y_{.j.}^2 - CF$$

$$\text{SS[Err1]} = \frac{1}{b}\sum_{i,j} Y_{ij.}^2 - \frac{1}{ab}\sum_i Y_{i..}^2 - \frac{1}{rb}\sum_j Y_{.j.}^2 - CF$$

SS[B] = $\frac{1}{ra}\sum_k Y_{..k}^2 - CF$

SS[Err2] = $\frac{1}{a}\sum_{i,k} Y_{i.k}^2 - \frac{1}{ab}\sum_i Y_{i..}^2 - \frac{1}{ra}\sum_k Y_{..k}^2 - CF$

SS[AB] = $\frac{1}{r}\sum_{j,k} Y_{.jk}^2 - \frac{1}{rb}\sum_j Y_{.j.}^2 - \frac{1}{ra}\sum_k Y_{..k}^2 - CF$

SS[Err3] = SS[Total] – SS[Reps] – SS[A] – SS[Err1] – SS[B] – SS[Err2]- SS[AB].

QR Decomposition could be used to find the sum of squares of its source of variation's components as long as the number of rows of the matrix is at least equal to the number of coloumns. Thus, in the simple strip-plot design QR decomposition can not be used whenever $rab < (1+r+a+ra+b+rb+ab)+1$. Partitioning the design matrix with respect to source of variation's component could help calculating their sum of squares and determining the rank of its partitioned matrices which are also their degrees of freedom.

The sum of squares of its source of variation's could be calculated from the last coloumn of matrix R. The square of the first row of this coloumn is for calculating the correction factor, the sum of the square of next *r* rows of this coloumn is for calculating the SS[Reps], the sum of the square of next *a* rows is for the calculating SS[A], and so on as long as $rab \geq (1+r+a+ra+b+rb+ab)+1$

3. LINEAR MODELS IN MATRIX NOTATION

A typical model considered is

$$\underline{Y} = X\underline{\beta} + \underline{e}$$

where $\underline{Y}$ is an *n x 1* vector of observations, X is an *n x p* matrix of known constants called the design matrix, $\underline{\beta}$ is a *p x 1* vector of unobservable parameters, and $\underline{e}$ is an *n x 1* vector of unobservable random errors. It is assumed that $E(\underline{e}) = 0$ and Cov($\underline{e}$) = $\sigma^2 I$.

The design matrix *X* above having size *rab x (1+r+a+ra+b+rb+ab),* is partitioned according to its source of variation components : Constants, Repetitions, Factor A, Error of Factor A, Factor B, Error of Factor B, and AB Interaction Effect.

Let $X_\mu = \underline{1}_{r\times 1} \otimes \underline{1}_{a\times 1} \otimes \underline{1}_{b\times 1}$ be the constant design matrix, $X_\rho = I_{r\times r} \otimes \underline{1}_{a\times 1} \otimes \underline{1}_{b\times 1}$ the repetition matrix, $X_\alpha = \underline{1}_{r\times 1} \otimes I_{a\times a} \otimes \underline{1}_{b\times 1}$ the main effect of factor A design matrix, $X_\delta = I_{r\times r} \otimes I_{a\times a} \otimes \underline{1}_{b\times 1}$ error for A design matrix, $X_\beta = \underline{1}_{r\times 1} \otimes \underline{1}_{a\times 1} \otimes I_{b\times b}$ the main effect of factor B design matrix, $X_\eta = I_{r\times r} \otimes \underline{1}_{a\times 1} \otimes I_{b\times b}$ error for B design matrix, and $X_{\alpha\beta} = \underline{1}_{r\times 1} \otimes I_{a\times a} \otimes I_{b\times b}$ the interaction effect of factor A and B design matrix, then $X = [X_\mu \mid X_\rho \mid X_\alpha \mid X_\delta \mid X_\beta \mid X_\eta \mid X_{\alpha\beta}]$.

Furthermore, the projection matrix has the form of $M_* = X_*(X_*^t X_*)^{-1} X_*^t$. Therefore, for each partitioned design matrix, it is easily to verify that the projection matrices with respect to each of the above design matrices are as follows :

$$M_\mu = X_\mu (X_\mu^t X_\mu)^{-1} X_\mu^t = \frac{1}{rab} J_{r\times r} \otimes J_{a\times a} \otimes J_{b\times b},$$

$$M_\rho = X_\rho (X_\rho^t X_\rho)^{-1} X_\rho^t = \frac{1}{ab} I_{r\times r} \otimes J_{a\times a} \otimes J_{b\times b},$$

$$M_\alpha = X_\alpha (X_\alpha^t X_\alpha)^{-1} X_\alpha^t = \frac{1}{rb} J_{r\times r} \otimes I_{a\times a} \otimes J_{b\times b},$$

$$M_\delta = X_\delta (X_\delta^t X_\delta)^{-1} X_\delta^t = \frac{1}{b} I_{r\times r} \otimes I_{a\times a} \otimes J_{b\times b},$$

$$M_\beta = X_\beta (X_\beta^t X_\beta)^{-1} X_\beta^t = \frac{1}{ra} J_{r\times r} \otimes J_{a\times a} \otimes I_{b\times b},$$

$$M_\eta = X_\eta (X_\eta^t X_\eta)^{-1} X_\eta^t = \frac{1}{a} I_{r\times r} \otimes J_{a\times a} \otimes I_{b\times b}$$

and

$$M_{\alpha\beta} = X_{\alpha\beta} (X_{\alpha\beta}^t X_{\alpha\beta})^{-1} X_{\alpha\beta}^t = \frac{1}{r} J_{r\times r} \otimes I_{a\times a} \otimes I_{b\times b}.$$

Using the properties of Kronecker product, we can easily find simple form every combination of matrix multiplication of M_μ, M_ρ, M_α, M_δ, M_β, M_η, and $M_{\alpha\beta}$. The results of these multiplications are summarized as in Table 1.

Table 1. Matrix Multiplication

	M_μ	M_ρ	M_α	M_δ	M_β	M_η	$M_{\alpha\beta}$
M_μ	M_μ	M_μ	M_μ	M_μ	M_μ	M_μ	M_μ
M_ρ	M_μ	M_ρ	M_μ	M_ρ	M_μ	M_ρ	M_μ
M_α	M_μ	M_μ	M_α	M_α	M_μ	M_μ	M_α
M_δ	M_μ	M_ρ	M_α	M_δ	M_μ	M_ρ	M_α

M_β	M_μ	M_μ	M_μ	M_μ	M_β	M_β	M_β
M_η	M_μ	M_ρ	M_μ	M_ρ	M_β	M_η	M_β
$M_{\alpha\beta}$	M_μ	M_μ	M_α	M_α	M_β	M_β	M_β

4. SUM OF SQUARES IN MATRIX NOTATION

In terms of matrix notation, the formulas for calculating sum of squares from strip-plot experiments as mentioned in **(2)**, can be written as follows:

SS[Reps] = $\underline{Y}^t(M_\rho - M_\mu)\underline{Y}$;

SS[A] = $\underline{Y}^t(M_\alpha - M_\mu)\underline{Y}$;

SS[Err1] = $\underline{Y}^t(M_\delta - M_\rho - M_\alpha + M_\mu)\underline{Y}$;

SS[B] = $\underline{Y}^t(M_\beta - M_\mu)\underline{Y}$;

SS[Err2] = $\underline{Y}^t(M_\eta - M_\rho - M_\beta + M_\mu)\underline{Y}$;

SS[AB] = $\underline{Y}^t(M_{\alpha\beta} - M_\alpha - M_\beta + M_\mu)\underline{Y}$ and

SS[Err3]=

$\underline{Y}^t(I - M_\mu + M_\rho + M_\alpha - M_\delta + M_\beta - M_\eta - M_{\alpha\beta})\underline{Y}$

Let the *n x 1* random vector $\underline{Y}$ be distributed **N($\underline{y}$: μ, I).** The random variable *U = Y'AY* is distributed as χ^2 (u:*K;λ*), where *λ* = ***μ'Aμ****/2*, if and only if ***A*** is an **idempotent** matrix of **rank *K*** [**5**].

It can be easily verified, using the result presented in Table 1 that $M_\rho - M_\mu$, $M_\alpha - M_\mu$, $M_\delta - M_\rho - M_\alpha + M_\mu$, $M_\beta - M_\mu$, $M_\eta - M_\rho - M_\beta + M_\mu$, $M_{\alpha\beta} - M_\alpha - M_\beta + M_\mu$, and $I - M_\mu + M_\rho + M_\alpha - M_\delta + M_\beta - M_\eta - M_{\alpha\beta}$ are all idempotent matrices. In additions to those idempotent properties, they are also **symmetric** matrices. From the properties of symmetric and idempotent matrices, their ranks are just equal to their **traces** [**6**]. Thus, their ranks are

tr($M_\rho - M_\mu$ *) = r-1,*

tr($M_\alpha - M_\mu$ *) = a-1,*

tr($M_\delta - M_\rho - M_\alpha + M_\mu$ *)=(r-1)(a-1),*

tr($M_\beta - M_\mu$ *) = b-1,*

tr($M_\eta - M_\rho - M_\beta + M_\mu$ *) =(r-1)(b-1),*

tr($M_{\alpha\beta} - M_\alpha - M_\beta + M_\mu$ *) = (a-1)(b-1)*

and

tr($I - M_\mu + M_\rho + M_\alpha - M_\delta + M_\beta - M_\eta - M_{\alpha\beta}$ *)*

= (r-1)(a-1)(b-1) respectively.

Using the above arguments, and without loss of generality that the random vector $\underline{Y}$ be distributed **N($\underline{y}$: $\underline{0}$, I),** therefore the distributions of sum of squares are as follows: SS[Reps] is distributed as χ^2 (*r-1*); SS[A] is distributed as χ^2 (*a-1*); SS[Err1] is distributed as χ^2 (*(r-1)(a-1)*); SS[B] is distributed as χ^2 (*b-1*); SS[Err2] is distributed as χ^2 (*(r-1)(b-1)*); SS[AB] is distributed as χ^2 (*(a-1)(b-1)*); and SS[Err3] is distributed as χ^2 (*(r-1)(a-1)(b-1)*).

5. HYPOTHESIS TESTING

Mean of Square is defined as Sum of Square divided by its degrees of freedom. Therefore, we have the followings : MS[Reps] = SS[Reps]/(*r-1*), MS[A] = SS[A]/(*a-1*), MS[Err1] = SS[Err1]/(*(r-1)(a-1)*), MS[B] = SS[B]/(*b-1*), MS[Err2] = SS[Err2]/(*(r-1)(b-1)*) MS[AB] = SS[AB]/(*(a-1)(b-1)*), and MS[Err3] = SSSP/(*(r-1)(a-1)(b-1)*).

We need to know first the expected mean squares (EMS) for each of the source of variation in the basic strip-plot design.

EMS[Reps]= $\sigma_\varepsilon^2 + b\sigma_\delta^2 + Y'(M_\rho - M_\mu)Y/(r-1)$,

EMS[A] = $\sigma_\varepsilon^2 + b\sigma_\delta^2 + Y'(M_a - M_\mu)Y/(a-1)$,

EMS[Err1] = $\sigma_\varepsilon^2 + b\sigma_\delta^2$,

EMS[B] = $\sigma_\varepsilon^2 + a\sigma_\eta^2 + Y'(M_\beta - M_\mu)Y/(b-1)$,

EMS[Err2] = $\sigma_\varepsilon^2 + a\sigma_\eta^2$,

EMS[AB]=

$\sigma_\varepsilon^2 + Y'(M_{\alpha\beta} - M_\alpha - M_\beta + M_\mu)Y/((a-1)(b-1))$

and EMS[Err3] = σ_ε^2.

We know that if A is distributed as chi-square with *a* degrees of freedom, B is distributed as chi-square with *b* degrees of fredom, A and B are independent to each other, then (A/*a*)/(B/*b*) is distributed as F with *a* and *b* degrees of freedom [**5**].

To check the independence of two matrices A and B, we need to show that AB = O [**7**]. Using the information in Table 1, it is easy to verify that $M_{\alpha\beta} - M_\alpha - M_\beta + M_\mu$ and $I - M_\mu + M_\rho + M_\alpha - M_\delta + M_\beta - M_\eta - M_{\alpha\beta}$ are independent; $M_\beta - M_\mu$ and $M_\eta - M_\rho - M_\beta + M_\mu$ are independent, $M_\alpha - M_\mu$ and $M_\delta - M_\rho - M_\alpha + M_\mu$ are independent; $M_\rho - M_\mu$ and $M_\delta - M_\rho - M_\alpha + M_\mu$ are independent; $M_\delta - M_\rho - M_\alpha + M_\mu$ and $I - M_\mu + M_\rho + M_\alpha - M_\delta + M_\beta - M_\eta - M_{\alpha\beta}$ are independent, also $M_\eta - M_\rho - M_\beta + M_\mu$ and

$I - M_{\mu} + M_{\rho} + M_{\alpha} - M_{\delta} + M_{\beta} - M_{\eta} - M_{\alpha\beta}$ are independent.

Therefore, we have the following results :

a. to test if there is a significant Reps effect is to reject the null hypothesis whenever $\frac{MS[\mathrm{Re}\,ps]}{MS[Err1]}$ is large enough. $\frac{MS[\mathrm{Re}\,ps]}{MS[Err1]}$ is distributed as F with *r*-1 and *(r-1)(a-1)* degrees of freedom.

b. to test if there is a significant A effect is to reject the null hypothesis whenever $\frac{MS[A]}{MS[Err1]}$ is large enough. $\frac{MS[A]}{MS[Err1]}$ is distributed as F with *a*-1 and *(r-1)(a-1)* degrees of freedom.

c. to test if there is a significant B effect is to reject the null hypothesis whenever $\frac{MS[B]}{MS[Err2]}$ is large enough. $\frac{MS[B]}{MS[Err2]}$ is distributed as F with *b*-1 and *(r-1)(b-1)* degrees of freedom.

d. to test if there is a significant AB interaction effect is to reject the null hypothesis whenever $\frac{MS[AB]}{MS[Err3]}$ is large enough. $\frac{MS[AB]}{MS[Err3]}$ is distributed as F with *(a-1)(b-1)* and *(r-1)(a-1)(b-1)* degrees of freedom.

6. EXAMPLE

As an illustration, to give an idea how to analyze the Strip-Plot experiments, the following example is taken from Lentner and Bishop, 1986.

A Turf specialist is studying the durability of six varietiesof turf grassin combinationwith three levels of compacting (none, slight, and moderate). Sufficient land was available at three locations (replicates) for use in the study. In each replicate, the turf varieties were established on six plots. The compacting machine could not be maneuvered easily within the whole plots established for varieties, so it was necessary to run the compacting machine in strips. The variable of interest was the amount of dry matter from a sample taken on each subunit. The results were (in grams):

Reps	Comp	Variety					
		1	2	3	4	5	6
1	1	10.3	9.7	11.2	10.8	10.5	9.9
	2	9.8	10.1	11.0	10.4	10.6	9.5
	3	9.0	9.6	10.8	10.1	9.8	11.0
2	1	11.8	10.3	12.1	12.3	11.8	10.6
	2	10.7	11.6	11.9	11.8	11.7	10.1
	3	10.1	10.9	12.1	11.0	10.3	9.2
3	1	10.2	10.1	11.6	11.2	10.6	10.3
	2	9.5	10.7	10.8	9.9	10.5	9.4
	3	9.7	9.3	11.2	9.6	10.4	10.3

R routine to analyze the above data is given below.

```
##### Given the Data #####
#### r is the number of replications
#### a is the level size of factor A
#### b is the level size of factor B
r <-3
a <-6
b <-3

##### Observed Data #####
y <-rbind(10.3, 9.8, 9.0, 9.7, 10.1, 9.6, 11.2,
11.0, 10.8, 10.8, 10.4, 10.1, 10.5, 10.6, 9.8,
9.9, 9.5, 11.0, 11.8, 10.7, 10.1, 10.3, 11.6,
10.9, 12.1, 11.9, 12.1, 12.3, 11.8, 11.0, 11.8,
11.7, 10.3, 10.6, 10.1, 9.2, 10.2, 9.5, 9.7, 10.1,
10.7, 9.3, 11.6, 10.8, 11.2, 11.2, 9.9, 9.6, 10.6,
10.5, 10.4, 10.3, 9.4, 10.3)

##### Basic Vectors and Matrices #####
vr <- matrix(1,r,1) #vektor 1r
va <- matrix(1,a,1) #vektor 1a
vb <- matrix(1,b,1) #vektor 1b
Ir <- diag(1,r,r) #identitas r
Ia <- diag(1,a,a) #identitas a
Ib <- diag(1,b,b) #identitas b

##### Partitioned Design Matrices #####
Xmu <- kronecker(vr,kronecker(va,vb))
Xr <- kronecker(Ir,kronecker(va,vb))
Xa <- kronecker(vr,kronecker(Ia,vb))
Xd <- kronecker(Ir,kronecker(Ia,vb))
Xb <- kronecker(vr,kronecker(va,Ib))
Xh <- kronecker(Ir,kronecker(va,Ib))
Xab <- kronecker(vr,kronecker(Ia,Ib))

##### Projection Matrices #####
Mmu <-(Xmu
%*%(solve(t(Xmu)%*%Xmu)))%*% t(Xmu)
Mr <-(Xr %*%(solve(t(Xr)%*%Xr)))%*%
t(Xr)
Ma <-(Xa %*%(solve(t(Xa)%*%Xa)))%*%
t(Xa)
Md <-(Xd %*%(solve(t(Xd)%*%Xd)))%*%
t(Xd)
```

```
Mb <-(Xb %*%(solve(t(Xb)%*%Xb)))%*% t(Xb)
Mh <-(Xh %*%(solve(t(Xh)%*%Xh)))%*% t(Xh)
Mab <-(Xab %*%(solve(t(Xab)%*%Xab)))%*% t(Xab)

##### Calculating Sum of Squares #####

SSReps <- round(t(y)%*%(Mr-Mmu)%*%y, digits=3)
SSA <- round(t(y)%*%(Ma-Mmu)%*%y, digits=3)
SSErr1 <- round(t(y)%*%(Md-Mr-Ma+Mmu)%*%y,digits=3)
SSB <- round(t(y)%*%(Mb-Mmu)%*%y, digits=3)
SSErr2 <- round(t(y)%*%(Mh-Mr-Mb+Mmu)%*%y,digits=3)
SSAB <- round(t(y)%*%(Mab-Ma-Mb+Mmu)%*%y,digits=3)
SSErr3 <- round(t(y)%*%(diag(1,r*a*b,r*a*b)-Mmu+Mr+Ma-Md+Mb-Mh-Mab)%*%y,digits=3)
SSTotal <- round(t(y)%*%(diag(1,r*a*b,r*a*b)-Mmu)%*%y,digits=3)

##### Calculating Means of Squares #####
library (psych)
MSReps <- round(SSReps/tr(Mr-Mmu),digits=3)
MSA <- round(SSA/tr(Ma-Mmu),digits=3)
MSErr1 <- round(SSErr1/tr(Md-Mr-Ma+Mmu),digits=3)
MSB <- round(SSB/tr(Mb-Mmu),digits=3)
MSErr2 <- round(SSErr2/tr(Mh-Mr-Mb+Mmu),digits=3)
MSAB <- round(SSAB/tr(Mab-Ma-Mb+Mmu),digits=3)
MSErr3 <- round(SSErr3/tr(diag(1,r*a*b,r*a*b)-Mmu+Mr+Ma-Md+Mb-Mh-Mab),digits=3)

##### Calculating F #####
FReps <- round(MSReps/MSErr1,digits=3)
FA <- round(MSA/MSErr1,digits=3)
FB <- round(MSB/MSErr2,digits=3)
FAB <- round(MSAB/MSErr3,digits=3)

##### Summary #####
sources <- rbind("Reps", "A","Err1","B","Err2","AB","Err3","Total")
Values <- cbind("Source","Deg Frdm","SS","MS","F")
SS1 <- rbind(SSReps,SSA,SSErr1,SSB,SSErr2,SSAB,SSErr3,SSTotal)
MS1 <- rbind(MSReps,MSA,MSErr1,MSB,MSErr2,MSAB,MSErr3,"")
DB1 <-rbind(tr(Mr-Mmu),tr(Ma-Mmu),tr(Md-Mr-Ma+Mmu),tr(Mb-Mmu),tr(Mh-Mr-Mb+Mmu),tr(Mab-Ma-Mb+Mmu),
        tr(diag(1,r*a*b,r*a*b)-Mmu+Mr+Ma-Md+Mb-Mh-Mab),tr(diag(1,r*a*b,r*a*b)-Mmu))
F1 <- rbind(FReps,FA," ",FB," ",FAB,"","")
stripplot <-cbind(sources,DB1,SS1,MS1,F1)
outstripplot <-rbind(Values,stripplot)
outstripplot
```

The above program results the following output.

```
     [,1]      [,2]       [,3]     [,4]     [,5]
[1,] "Source" "Deg Fre"  "SS"     "MS"     "F"
[2,] "Reps"   "2"        "9.053"  "4.527"  "16.110"
[3,] "A"      "5"        "12.191" "2.438"  "8.676"
[4,] "Err1"   "10"       "2.811"  "0.281"  " "
[5,] "B"      "2"        "3.301"  "1.651"  "7.116"
[6,] "Err2"   "4"        "0.929"  "0.232"  " "
[7,] "AB"     "10"       "4.450"  "0.445"  "2.587"
[8,] "Err3"   "20"       "3.440"  "0.172"  " "
[9,] "Total"  "53"       "36.175" " "      " "
>
```

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